

# Stable Matching / Analyzing Efficiency

# Today

- **Announcement:** HW1 is posted, due next Thursday in class
- Topics
  - Stable matching proofs
  - Efficiency of algorithms

# Propose-and-Reject (Gale-Shapley) Algorithm

```
Initialize each college and student to be free.
while (some college is free and hasn't made
offers to every student) {
  Choose such a college c
  s = 1st student on c's list to whom c has not
  made offer
  if (s is free)
    assign c and s to be engaged
  else if (s prefers c to current college c')
    assign c and s to be engaged, and c' to be
    free
  else
    s rejects c
}
```

# Questions about the Gale-Shapley Algorithm

- ✓ ● Does the loop terminate?
- Is the matching perfect, that is, is it one-to-one?
- Is the matching stable?

# Proof by Contradiction (Review)

• Goal: prove that  $A$  is true

1. **Assume**  $A$  is **false**.
2. **Reason to a contradiction** with some other known fact
3. **Conclude** that  $A$  must therefore be **true**.

# On Fairness...

- Gale-Shapley algorithm is asymmetric: seems that colleges have the upper hand
- What can we say formally?

# A Remarkable Result

- A given problem instance may have several stable matchings

**Def.** College  $c$  is a **valid partner** of student  $s$  if there exists some stable matching in which they are matched.

**College-optimal assignment.** Each college receives best valid student.

**Claim.** All executions of GS yield **college-optimal** assignment, which is a stable matching!

What proof technique should we use?

# Algorithm Design

- ✓ • Formulate the problem precisely
- ✓ • Design an algorithm to solve the problem
- ✓ • Prove the algorithm correct
- Analyze the algorithm's runtime

# Analyzing Running Time (Chapter 2)

- What is efficiency?
- Tools: asymptotic growth of functions
- Practice finding asymptotic running time of algorithms

# Is My Algorithm Efficient?

- Idea: Implement it, time how long it takes.

- Problems?

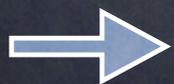
- Effects of the programming language?

- Effects of the processor?

- Effects of the amount of memory?

- Effects of other things running on the computer?

- Effects of the input values?



- Effects of the input size?